# Coulomb interactions in the intracluster medium

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### ABSTRACT

In this paper we discuss the effect of Coulomb collisions on the temperature profiles of the intracluster medium in clusters of galaxies, motivated by recent reports of negative temperature gradients in some clusters by Markevitch et al. The timescale for electrons and protons to reach temperature equilibrium can exceed a few billion years beyond radii of a Mpc, if the intracluster gas is assumed to be at the usual cluster virial temperature. If a cluster merger has occurred within that time causing the protons, but not the electrons, to be rapidly heated then a small negative temperature gradient can result. This gradient is larger in clusters with high temperatures and steep density profiles.

Applying these considerations to the cluster of galaxies A2163, we conclude that, more plausibly, the observed gradient is due to a lack of hydrostatic equilibrium following a merger.

**Key words:** X-ray: galaxies – galaxies: clusters: general – galaxies: clusters: individual: A2163

### 1 INTRODUCTION

Recent results on steep temperature profiles in some clusters of galaxies, as measured by ASCA X-ray spectra (Markevitch 1996 and Markevitch et al. 1996), have raised questions on the physical condition of the intracluster medium (ICM). At a radius of a few 100 kpc, the ICM has a characteristic density  $n_{\rm gas} \sim 10^{-3}~{\rm cm}^{-3}$ , temperature  $T_{\rm gas} \sim 10^8~{\rm K}$  and a heavy element abundance of about 30 per cent of the solar value. The density drops at larger radii r approximately as  $r^2$ .

Assuming a polytropic distribution for the gas, the temperature scales with density as

$$\frac{T_{\text{gas}}}{T_0} = \left(\frac{n_{\text{gas}}}{n_0}\right)^{\gamma - 1},\tag{1}$$

where the polytropic index  $\gamma$  ranges between 1 and 5/3, the limits corresponding to the gas being isothermal and adiabatic, respectively. Markevitch (1996) found that  $\gamma\approx 1.9$  and 1.7 for the clusters A2163 and A665, respectively (> 1.7 and 1.3 and the 90 per cent confidence level). When  $\gamma>5/3$  the gas is convectively unstable, which tends to produce turbulent motions in the ICM, resulting in a mixed, homogeneous gas after several sound crossing times. The detection of a dramatic drop in the temperature profile may be an indication that those clusters are observed in a very unusual, brief stage in their existence, perhaps having experienced a major merger within the previous few billion years.

An alternative possibility to explain the steep temperature decline, suggested by Markevitch (1996 and Markevitch et al 1996), is that the electron temperature,  $T_{\rm e}$ , which is the

quantity measured by X-ray observations, is not representative of the mean gas temperature,  $T_{\rm gas}$ . This can occur, for example if a shock has heated the ICM, with the protons receiving most of the energy so that the proton temperature  $T_{\rm p}$  greatly exceeds the electron temperature  $T_{\rm e}$ . Coulomb scattering then equilibrates the temperatures at a rate (Spitzer 1962):

$$\frac{dT_{\rm e}}{dt} = \frac{T_{\rm p} - T_{\rm e}}{t_{\rm eq}}.$$
 (2)

In this equation,  $t_{\rm eq}$ , the equipartition time via Coulomb scattering, is given by

$$t_{\rm eq} = \sqrt{\frac{\pi}{2}} \frac{m_{\rm p}}{m_{\rm e}} \frac{\theta_{\rm e}^{3/2}}{n_{\rm p} c \sigma_{\rm T} \ln \Lambda},\tag{3}$$

where  $\theta_{\rm e} = k_{\rm B}T_{\rm e}/(m_{\rm e}c^2)$ ,  $n_{\rm p}$  is the proton density,  $\sigma_{\rm T}$  is the Thomson scattering cross section and  $\Lambda$  is the ratio of largest to smallest impact parameters for the collisions ( $\ln \Lambda \sim 37.8$ , Sarazin 1988; note that a similar term in the corresponding dimensionless proton temperature  $\theta_{\rm p}$  is negligible for the conditions of the ICM). The rate is proportional to the gas density and so can be low, and the equilibration time long, at the outer parts of a cluster where the density is least.

Here we examine the equilibration timescale for a typical ICM under the conservation of total kinetic energy. We find (Section 2) that an implausibly short timescale is required for a large temperature gradient to be observed. Both the X-ray emission and the equilibration of the ICM depend upon Coulomb scattering, so when the rate of emission is high the rate of equilibration is also high. In Section 3, we

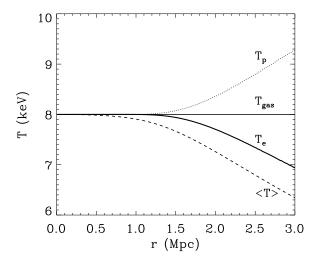


Figure 1. This plot shows the temperature profiles as a function of radius r at the time  $t_{\rm m}$  of 3 Gyrs. The input parameters are:  $n_0=2.5\times 10^{-3}{\rm cm}^{-3}, \beta=2/3$  and  $r_{\rm c}=0.3$ . The thick solid line indicate the effect of the e–p Coulomb collisions on the electron temperature when the energy density U is conserved and assuming that  $T_{\rm gas}(r)$  is isothermal. The radial decrease of  $T_{\rm e}$  with respect to the central value is about 0.02, 4 and 14 per cent at 1, 2 and 3 Mpc, respectively. The dotted line describes the  $T_{\rm p}$  profile and the dashed line the projected temperature < T >. The latter profile diminishes with respect to the central value by 1, 9 and 21 per cent, at 1, 2 and 3 Mpc, respectively.

apply these considerations to the case of A2163 and discuss our conclusions in Section 4.

## 2 ELECTRON-PROTON COULOMB INTERACTIONS

The accretion of infalling material on the cluster potential shock heats the protons to an initial temperature  $T_{\rm p}^i$ , which is representative of their isotropic Maxwellian velocity distribution. The protons equilibrate between themselves in a time shorter than  $t_{\rm eq}$  by a factor  $(m_{\rm e}/m_{\rm p})^{1/2} \sim 0.02$ , so that they can be considered to be in equilibrium when the heating event occurred a time  $t_{\rm m} \sim t_{\rm eq}$  in the past. The electrons, which are not strongly involved in shock events due to their negligible mass, achieve a low temperature  $T_{\rm e}^i \ll T_{\rm p}^i$ .

Since the total kinetic energy density  $U = \frac{3}{2}k(n_{\rm p}T_{\rm p} + n_{\rm e}T_{\rm e})$  has to be conserved, we have the following implications: (i) the local mean gas temperature,  $T_{\rm gas}$ ,

$$T_{\rm gas} = \frac{n_{\rm p}T_{\rm p} + n_{\rm e}T_{\rm e}}{n_{\rm gas}} = \frac{T_{\rm p} + 1.21T_{\rm e}}{2.21}$$
 (4)

is constant with time; (ii) the initial  $T_{\rm p}^i$  is about 2.2 times the balanced temperature value,  $T_{\rm gas}^i$ ; (iii)  $T_{\rm e}$  increases (and  $T_{\rm p}$  decreases) with time, with the energy exchange between protons and electrons driven by the relation

$$\frac{dT_{\rm e}}{dt} = -(n_{\rm p}/n_{\rm e})\frac{dT_{\rm p}}{dt}.\tag{5}$$

Rearranging eqn.(2) in the form:

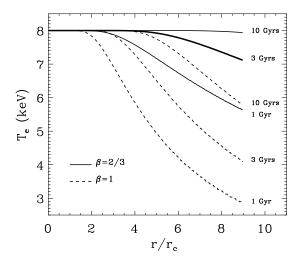


Figure 2. The behavior of the electron temperature as function of  $r/r_{\rm c}$ , when  $\beta=2/3$  (solid line) and  $\beta=1$  (dashed line). The different profile were calculated with different  $t_{\rm m}$  (upwards: 1, 3, 10 Gyrs), assuming constant  $n_0$  and fixing  $T_{\rm e}(r=0)$  at 8 keV. The thickest solid line corresponds to  $\beta=2/3$  and  $t_{\rm m}=3$  Gyrs.

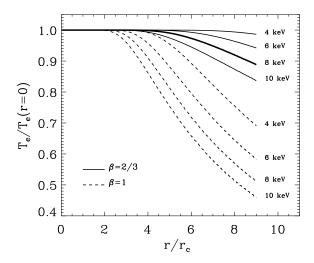
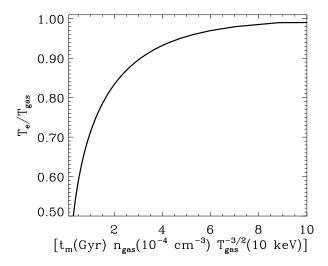


Figure 3. The dependence of  $T_{\rm e}$ , normalized to the central value  $T_{\rm e}(r=0)$ , upon different central temperature values (downwards: 4, 6, 8 – the thickest line – and 10 keV) is shown. All the profiles were calculated at  $t_{\rm m}=3$  Gyrs, assuming  $\beta=2/3$  (solid lines) and  $\beta=1$  (dashed lines).

$$\int_{0}^{T_{\rm e}} \frac{t_{\rm eq}}{T_{\rm p} - T_{\rm e}} dT_{\rm e} = \int_{0}^{t_{\rm m}} dt, \tag{6}$$

and using relation (4), we can solve analytically eqn.(6), obtaining

$$\ln\left(\frac{\sqrt{T_{\rm gas}} + \sqrt{T_{\rm e}}}{\sqrt{T_{\rm gas}} - \sqrt{T_{\rm e}}}\right) - \frac{2}{3} \left(\frac{T_{\rm e}}{T_{\rm gas}}\right)^{3/2} - 2 \left(\frac{T_{\rm e}}{T_{\rm gas}}\right)^{1/2} = \sqrt{\frac{2}{\pi}} \frac{m_{\rm e}}{m_{\rm p}} \left(\frac{m_{\rm e}c^2}{k_{\rm B}T_{\rm gas}}\right)^{3/2} c\sigma_{\rm T} \ln\Lambda \ t_{\rm m} n_{\rm gas}. \tag{7}$$



**Figure 4.** The dependent variable  $T_{\rm e}/T_{\rm gas}$  at first member in eqn.(7) is plotted versus the independent variable present in the second member.

Where  $T_{\rm e}/T_{\rm gas} < 0.5$ , such as in the outer part of the temperature profile given a high central temperature and steep density profile, we can further expand the logarithmic term in eqn.(6). Then, for  $T_{\rm gas}$  =constant, we obtain that  $T_{\rm e} \propto n_{\rm gas}^{2/5}$ .

We now parametrise the proton density by a  $\beta$ -model (Cavaliere & Fusco-Femiano 1978), as generally adopted for the description of the ICM with central density  $n_0$  and core radius  $r_c$ ,

$$n_{\rm p}(r) = n_0 \left[ 1 + (r/r_{\rm c})^2 \right]^{-3\beta/2},$$
 (8)

and fix the electron density  $n_{\rm e} = 1.21 \times n_{\rm p}$ .

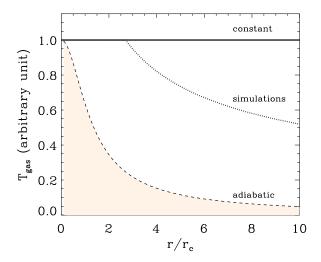
Results for reference values of  $t_{\rm m}=3$  Gyrs,  $T_{\rm e}(r=0)=8$  keV,  $n_0=2.5\times 10^{-3}$  cm<sup>-3</sup>,  $r_{\rm c}=0.3$  Mpc and  $\beta=2/3$ , are shown in Fig. 1. A significant small temperature gradient can remain at this reference time. In Fig. (1), we also show the projected temperature, < T>, which is the emissivity-weighted  $T_{\rm e}$ :

$$\langle T \rangle = \int_{b^2}^{\infty} \frac{\epsilon(r) \ T_{\rm e}(r) \ dr^2}{\sqrt{r^2 - b^2}} / \int_{b^2}^{\infty} \frac{\epsilon(r) \ dr^2}{\sqrt{r^2 - b^2}},$$
 (9)

where the volume emissivity  $\epsilon$  is integrated along the line of sight at projected radius b. It is more appropriate to compare with observed values and is shown to decrease more significantly than  $T_{\rm e}(r)$ .

The dependence of the temperature gradient  $T_{\rm e}(r)$  on  $t_{\rm m}$ ,  $\beta$  and the central value of  $T_{\rm e}$ ,  $T_{\rm e}(r=0)$ , is shown in Fig. 2 and 3. Note that the temperature gradient steepen when there is (i) a higher energy per particle (i.e. when a larger central  $T_{\rm e}$  is considered); (ii) a greater value for  $\beta$ , so that the gas density becomes steeper; or, of course, (iii) the timescale is shorter.

In Fig. 4, we show how  $T_{\rm e}/T_{\rm gas}$  varies with  $t_{\rm m}n_{\rm gas}T_{\rm gas}^{-3/2}$ . A 10 per cent difference in  $T_{\rm e}$ , with respect to  $T_{\rm gas}$ , requires that  $n_{\rm gas} < 3.09 \times 10^{-4} t_{\rm m}^{-1} T_{\rm gas}^{3/2} {\rm cm}^{-3}$ , where  $t_{\rm m}$  is in Gyrs and  $T_{\rm gas}$  in units of 10 keV. We note that the outer densities detectable by the ROSAT PSPC in deep cluster images are of order of about  $10^{-4} {\rm cm}^{-3}$ .



**Figure 5.** The temperatures profiles discussed in Sect. 2.1 are plotted here, assuming (i) a ratio  $r_{\rm c}/r_{200}$  of 0.15 (cf. table 1 in Navarro et al. 1996) for the dotted profile from the N-body simulations, and (ii) a  $\beta$ -model with  $\beta=2/3$  for the adiabatic profile (dashed line). The shadowy region indicates where the gradients cause the ICM to be convectively unstable.

# 2.1 The presence of a gradient in the gas temperature

Above, we have considered the case of  $T_{gas}(r) = constant$ .

However, recent N-body simulations of clusters of galaxies (Navarro et al. 1996, Evrard et al. 1996) have shown that  $T_{\rm gas}$  is almost constant in the range 0.1–0.4  $r/r_{200}$  (where  $r_{200}$  indicates the radius at which the cluster mean density is 200 times the critical density value) and slightly decreases as  $r^{-1/2}$  towards the outer virialized part. Such a gradient is not unstable to convective mixing, from the adiabatic condition [cf. eqn.(1) and Fig. 5]:

$$\frac{dT_{\rm gas}}{dr} > \frac{d\left(n_{\rm gas}^{2/3}\right)}{dr} \ . \tag{10}$$

We can use eqn.(7) (obtained under the condition of  $T_{\rm gas}$  constant with time) to investigate the more extreme electron temperature gradient when the gas is adiabatic. We obtain a behavior of  $T_{\rm e}$  almost coincident with the  $T_{\rm gas}$  profile shown in Fig. 5, for every  $t_{\rm m}$  considered.

## 3 THE CASE OF A2163

A long Coulomb equilibration timescale between electrons and protons has been suggested by Markevitch et al. as one explanation for the steep negative temperature profile observed in ASCA observations of A2163. We now try to test this possibility by comparing our calculated profile with the deprojected temperature in Abell 2163 (Markevitch et al. 1996; Elbaz et al. 1995; Fig. 6). In particular, we find that it is impossible to reconcile the theoretical temperature gradients with the observed one in the outer parts of the cluster, for any reasonable  $t_{\rm m}$  (even for 1 Gyr the disagreement is about of 5.4  $\sigma$ ), if the final state for the ICM is isothermal.

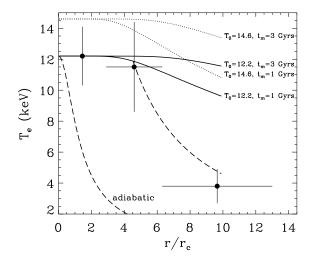


Figure 6. The temperature profile of A2163 from the analysis by Markevitch et al. (1996). The model profiles are calculated using as input the values in Elbaz et al. (1995):  $n_0 = 6.65 \times 10^{-3} \text{ cm}^{-3}$ ,  $\beta = 0.62$ .  $T_{\rm e}(r=0)$  is considered both equal to 12.2 (solid lines; Markevitch et al.) and 14.6 (dotted lines; Elbaz et al.) keV. The results from an assumed isothermal profile clearly fail at the outer radius. The two adiabatic profiles (dashed lines) are calculated at  $r/r_{\rm c}$  equal to 0 and 4.5 via eqn.(1) and with  $\gamma = 5/3$ . From the discussion in Sect. 2.1, they do not depend significantly on  $t_{\rm m}$ .

Only if we make the *ad hoc* assumption that the ICM is adiabatic beyond about one Mpc can we obtain some agreement (cf. dashed lines in Fig. 6).

We now consider whether the gas can be considered to be in hydrostatic equilibrium since the last plausible merger (a few billion years). The time required for a sound wave to cross a cluster is obtained by the relation  $v_{\rm S}^2 = dP_{\rm gas}/d\rho_{\rm gas}$ , where  $P_{\rm gas}$  and  $\rho_{\rm gas}$  are the ICM pressure and density, respectively. Solving this relation with respect to time, we get  $t_{\rm S} \propto r \, T_{\rm gas}^{-0.5}$ , that does not depend significantly on any temperature (density) gradient. Using the values in Fig. 6 for A2163, we calculate that  $t_{\rm S}$  is  $0.6\pm0.6, 2.1\pm0.8$  and  $7.7\pm2.9$ Gyrs at 0.45, 1.43 and 2.99 Mpc, respectively (the errors are  $1\sigma$  deviation obtained by propagation on r and  $T_{\rm gas}$  uncertainties). A critical condition for the hydrostatic assumption appears above 1.4 Mpc ( $\sim 5r_{\rm c}$ ), where the steep temperature profile requires that a merger occurred less than about 3 Gyrs ago, if the steepness is due to the Coulomb process. Then, the cluster appears unrelaxed in its outer parts, with the subsequent presence of gas bulk motions, whose pressure is not accounted for by the measured ICM temperature in the last spatial bin of Fig. 6 (see also the discussion in Markevitch et al. 1996). We conclude that in regions of  $(r, t_{\rm m})$  space where the electron temperature is significantly below the proton temperature then it is likely that hydrostatic equilibrium has broken down.

Another timescale relevant to the thermalization of the ICM is the ionization equilibrium time of the abundant elements,  $t_{\rm ion}$ . The emission lines of the highly ionized Fe atoms are the strongest features detected in numerous X-ray spectra of clusters of galaxies. Using the fitted parameters in Shull & Van Steenberg (1982) for the collisional ionization rate coefficients  $C_{\rm Fe}$  of the hydrogenic FeXXVI ions, we get

 $t_{\rm ion} = (C_{\rm Fe} \; n_{\rm e})^{-1} \sim 0.1-10$  Gyrs in the range  $0 < r/r_{\rm c} < 10$  for the density profiles considered above (i.e.  $\beta = 2/3$  and 1,  $T_{\rm e} = 8$  keV). In the case of A2163,  $t_{\rm ion}$  is of the order of a few tenth of Gyr, and less than 0.4 Gyr, which is shorter than any other timescale discussed here.

Finally, we note that the thermal conduction timescale on which temperature gradients are erased are much longer than the other timescales considered here by more than one order of magnitude.

### 4 CONCLUSIONS

In this paper, we have shown that electron–proton Coulomb interactions in ICM become inefficient in reaching the equipartition, steepen the electron temperature gradient significantly, only if (i) the energy per particle is high, (ii) the gas density profile is steep, and (iii) the time elapsed since the last merger, or proton heating event, is very short, i.e.  $n_{\rm gas} < 3.09 \times 10^{-4} (t_{\rm m}/1{\rm Gyr})^{-1} (T_{\rm gas}/10{\rm keV})^{3/2}{\rm cm}^{-3}$ . Local conservation of energy means that the proton temperature drops as the electron temperature rises. Regions of clusters where a large disequilibrium occurs are likely to both be out of hydrostatic equilibrium and to have low X-ray emission.

Similar conclusions are also presented by Fox & Loeb (1997), who consider Coulomb interactions in a plasma accreting on a cluster in the framework of the spherical self-similar model.

When applied to A2163, for which Markevitch et al. (1996) report a steep drop in electron temperature in the outer parts of the cluster, we find that we cannot reproduce the profile from gas in hydrostatic equilibrium, without requiring the mean gas temperature to drop sharply beyond about one Mpc.

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